PHYSICAL REVIEW E VOLUME 55, NUMBER 5 MAY 1997

Randomly pinned landscape evolution

Guido Caldarelli, Achille Giacometti, and Amos Maritan
Istituto Nazionale di Fisica della Materia, International School for Advanced Studies and INFN, via Beirut 2-4, Trieste I-34013, Italy

Ignacio Rodriguez-Iturbe
Department of Civil Engineering, Texas A&M University, College Station, Texas 77843

Andrea Rinaldo
Istituto di Idraulica ''G. Poleni,'' Università di Padova, I-35131 Padova, Italy

(Received 4 March 1997)

A simple scheme for the evolution of a fluvial landscape in heterogeneous environments is critically examined to capture the essential mechanism responsible for the recurrent scale-free landforms in the river basin. It is shown that, regardless of boundary and initial conditions, geomorphological constraints in the form of quenched randomly pinned regions play a key role in the robust emergence of aggregation patterns with a scaling behavior in agreement with that of real river basins. [S1063-651X(97)51805-8]

PACS number(s): 05.40.+j, 68.70.+w

Considerable efforts have been recently devoted to models of landscape evolution [1–3] in view of newly acquired experimental evidence showing recurrent scale-free structures in the geomorphology of the fluvial basin often covering over four orders of magnitude of linear size [4]. As in standard critical phenomena, the mechanism producing such structures is expected to depend only on a few basic features common to all the networks, rather than on the details of the particular system under consideration. This can be conveniently shown by classifying the results into a limited number of universality classes determined by a set of critical exponents.

A crucially simple lattice model, inspired by self-organized critical [5] models, aimed at the basic mechanism for fluvial landform evolution was recently proposed [6]. The model addresses only fluvial dynamics (whose imprinting is argued to dominate the morphology of river basins [1]) and is based on a dynamic threshold for activity defined by the exceeding of well-defined shear stresses.

Although the dynamics of evolution is purposely extremely simple, it has been shown [6] that this choice is both physically meaningful and suitable as an algorithmic searching mechanism.

Here we show that the results of the original simulations [6] were affected by finite size effects and by a limited choice of boundary and initial conditions which all combined to yield results very close to the experimental ones. Here we use larger scale simulations and more general boundary and initial conditions. In this way we identify two distinct universality classes. The first one, corresponding to the original formulation [6], fails to reproduce consistently the scaling behavior of field observations. The second one agrees with observational data.

Since in fluvial processes inhomogeneous precipitations and geomorphological constraints are expected to play a definite role in the development of the fluvial basin [7], we show that an essential ingredient of the model should effectively take heterogeneity of the terrain into account, a feature which was missing in the original model and which provides a new universality class.

Specifically we considered two physically based kinds of randomness. We have explicitly verified that the first one, namely heterogeneous rainfall, is not effective in changing the universality class of the original model. The second one was devised to model the geomorphological constraints in the form of random pinning of the landscape. This paper hinges on this feature which proves effective and of much physical significance and indeed defines the second class which consistently agrees with the one of real river basins.

Our lattice model is defined as follows. Let h_x be the height of the landscape associated with every site x of a square lattice of size $L\times L$. We assume that on the lowest side (kept at height $h=0$) all sites are possible outlets (multiple outlets) of an ensemble of rivers which are competing to drain the whole $L\times L$ basin. A random noise is added to the initial substrate in order to have unbiased initial conditions. Each site (pixel) collects water from a distributed injection in addition to the flow which drains to the site from the upstream connected sites. The steepest descent constructions allow the assignment of drainage directions to every site of an arbitrary landscape. The drained area a_x (which is also a measure of the total flow rates collected at that site) is associated with each site x according to the equation

$$a_x=\sum_{y(x)} a_y+r_x \qquad (1)$$

where the sum runs over the subset of neighbor sites $y(x)$ whose area is actually drained by x. If no contribution to x exists, then $a_x=r_x$ and it represents a source. The second term in Eq. (1) represents rainfall at position x. For uniform injection $r_x=1$. Random injection will be briefly discussed at the end. Another significant morphological indicator is the (up)stream length l_x from any site x to its source, which is computed according to straightforward procedures [8].

The time evolution of the model follows the following steps.

(1) The shear stress τ_x acting on every site is computed according to [6] $\tau_x = \Delta h_x a_x^{1/2}$ where Δh_x is the local gradient along the drainage direction.

(2) If the shear stress on a site exceeds a threshold value τ_c, then the corresponding height h_x is reduced—mimicking erosive mechanisms—in order to decrease the local gradient and the shear stress and set it just at the critical threshold. This produces a rearrangement of the network according to Eq. (1) followed by a new update of the whole pattern as in step (1).

(3) When all sites have shear stress below (or at) threshold, the system is in a dynamically steady state. Since this situation is not necessarily the most stable, a perturbation is applied to the network to increase the stability of a new steady state. A site is picked up at random and its height is increased in such a way that no lakes, i.e., sites whose height is lower than the heights of all neighbors, are formed. Steps (1) and (2) then follow as before.

A small percent p (typically 5%) of the heights are chosen (randomly and in uncorrelated manner) to be pinned to their initial value. This is equivalent to assigning a large value of τ_c (infinite in our case) to a randomly chosen fraction p of the sites and a value $\tau_c = 1$ to the remaining sites. Thus these sites do not take part in the landscape evolution process. Note that this is a reasonable geological feature (see Montgomery and Dietrich in Ref. [4]). One expects this effect to increase the meandering of the main river structures thus providing an additional mechanism for aggregation. The case $p = 0$ corresponds to the case studied in Ref. [6]. It is worth mentioning that for a given concentration p of pinned sites, a typical correlation length $l_p \sim p^{-1/2}$ (in two dimensions) is introduced. For system sizes $L \gg l_p \gg 1$ (=lattice spacing ~ 10 m), crossover from the pure ($p = 0$) case to the truly asymptotic behavior ($p \neq 0$, $L \to \infty$) should be observable. This is quite typical in disordered systems and it is observed here too.

After a suitable number of the perturbations [step (3)], the system reaches a steady state which is insensitive to further perturbations and where all statistics of the networks are

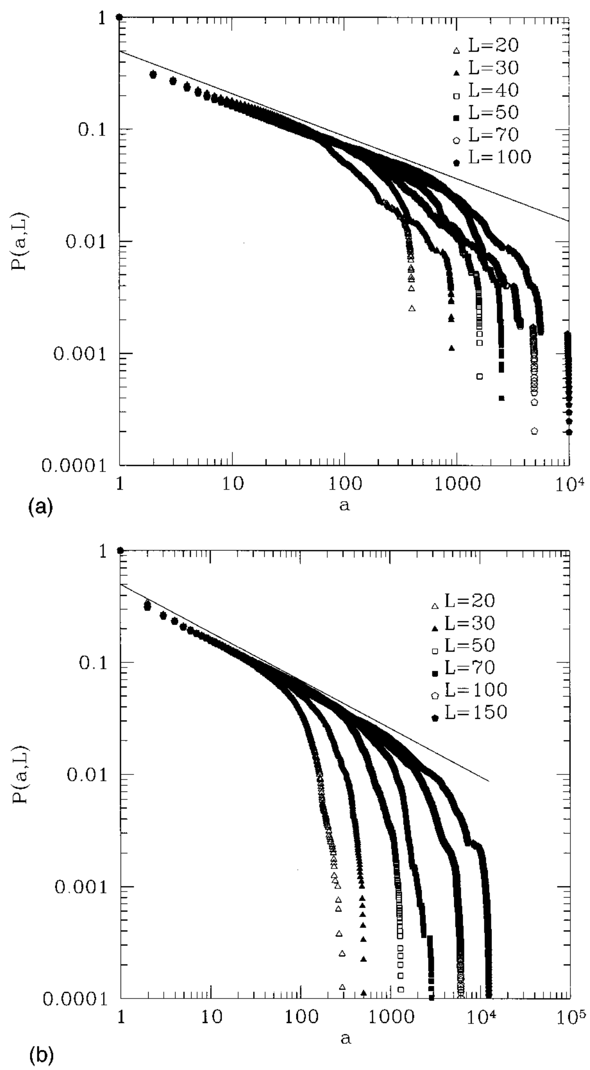

FIG. 1. Log-Log plot of the cumulative area distribution $P(a,L)$ versus a without (up) and with (down) pinning. The full line (slightly shifted upward for sake of clarity) has a slope corresponding to $\tau = 1.38$ and $\tau = 1.43$, respectively.

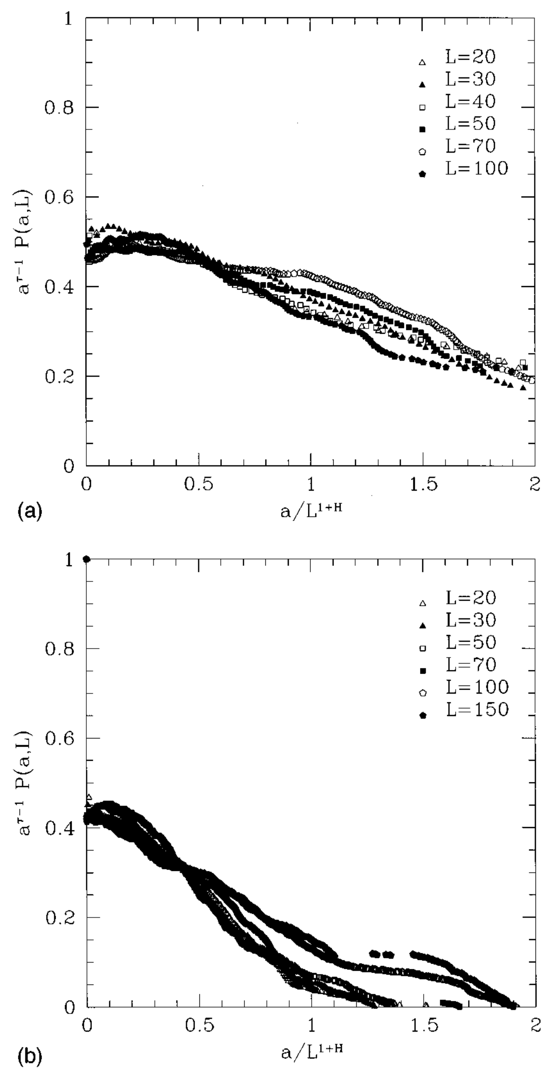

FIG. 2. Scaling function $a^{1-\tau}P(a,L)$ versus a/L^{1+H} in the same order as above. The values used to obtain the collapse were $\tau = 1.38$, $H = 0.60$ and $\tau = 1.43$, $H = 0.75$, respectively.

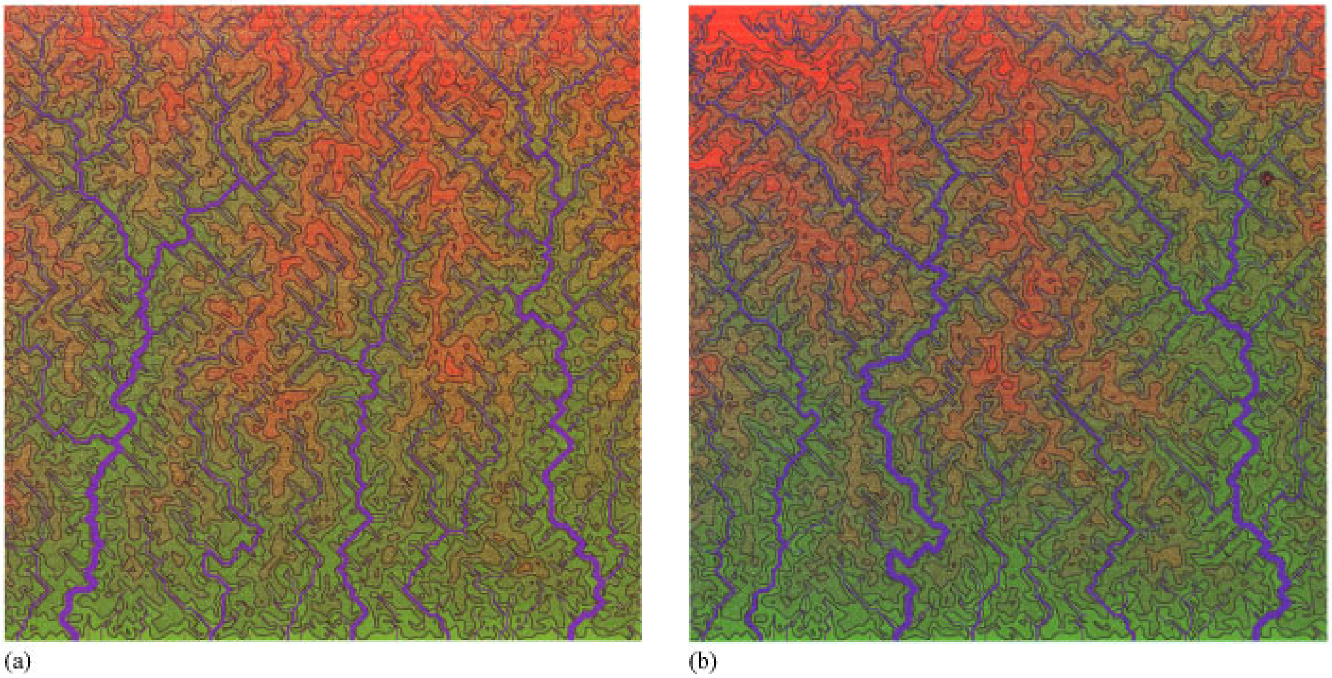

FIG. 3. (Color). Comparison between the evolution of two identical initial configurations of the model with size $L=100$ without (a) and with pinning (b). The pinning dilution was 5%.

stable. The resulting steady state proves scale-free, i.e. it is characterized by power-law distributions of the physical quantities of interest, which will be analyzed below.

Our numerical calculations were carried out on a two-dimensional square lattice (where each site has eight nearest neighbors) for sizes up to $L=200$ and a number of perturbations up to 10^6. This high number of perturbations involved makes the simulations very time consuming, thus preventing tests on much larger sizes. For comparison the original simulations [6] were carried out up to $L=64$.

The initial height field has a nonzero average slope m, along the ''main flow'' direction, and height fluctuations not exceeding 10% of mL. Boundary conditions are reflecting in the direction transversal to the main flow and open (multiple outlets are allowed) in the parallel one. Averages over few (up to five) configurations were routinely taken [9]. This choice proves sufficient in view of the self-averaging nature of the random perturbation.

The scaling analysis of the resulting landforms is based on $P(a,L)$ and $\Pi(l,L)$, the cumulative probability distribution of the drainage area a and stream length l, respectively, in a domain of linear size L. The following scaling forms are expected [8] to hold:

$$P(a,L)=a^{1-\tau}F\left(\frac{a}{L^{1+H}}\right), \tag{2}$$

$$\Pi(l,L)=l^{1-\gamma}G\left(\frac{l}{L^{d_l}}\right). \tag{3}$$

Here H is the Hurst (or wandering) exponent and d_l is the stream-length (or chemical distance) fractal exponent [10]. τ and γ are two of the various critical exponents that can be defined for river networks [1]. For self-affine river basins ($H<1$, $d_l=1$) scaling relations can be derived [8] yielding all exponents in terms of H only [11]. One finds

$$\tau=\frac{1+2H}{1+H}, \quad \gamma=1+H. \tag{4}$$

Experimental values of τ and γ are available from earlier experimental analyses from basins of different size, geology, exposed lithology, climate and vegetation [1]. There it was observed that, while a majority of basins tend to seemingly universal values $\tau=1.43\pm0.02$ and $\gamma=1.8\pm0.1$, exceptions are found where altered values are observed, i.e., the values change in a concerted manner and scaling relations such as those in Eq. (4) are still satisfied.

Scaling analysis for the cumulative area distributions are shown in Figs. 1(a) and 1(b). Figure 2 contains the corresponding collapse plots according to Eq. (2). Similar results are also obtained for the upstream lengths cumulative distributions. Our estimates of the critical exponents are independently derived and they agree with the scaling laws (4). Their values are $\tau=1.38\pm0.02$, $H=0.60\pm0.05$, and $\gamma=1.60\pm0.02$ in the absence of heterogeneity, i.e., $p=0$; $\tau=1.43\pm0.02$, $H=0.75\pm0.05$, and $\gamma=1.70\pm0.02$ for the random pinning case (i.e., $p\neq0$). The errors are statistical for τ and γ. For H the error is based on a careful estimate of the collapse plot. It should be noted that the collapse in Fig 2(b) is worse for the two highest sizes considered (see Fig. 3) due to poorer statistics.

Of course the fact that $H<1$ implies that $d_l=1$ has been used in Eq. (3) for the analysis of the upstream lengths distribution. However, corrections to scaling appear to be stronger for the lengths rather than the areas distribution. In Fig. 3 the typical network obtained using the above dynamics is reported without (a) and with (b) random pinning starting with the *same* initial configuration. It is worth noticing that the randomness increases the river wandering and thus the value of H and τ. The following remarks are in order.

(a) We have explicitly verified that our results do not depend on the imposed boundary conditions, i.e., changing from single to multiple outlets does not affect the critical exponents. This was the basic flaw in the original testing [6] of the model.

(b) When multiple outlets are allowed the statistics based on the largest rivers give slightly low Hurst exponents both for the homogeneous and the heterogeneous case. In all cases the resulting exponents are consistent with experimental values and with the scaling laws (4) within the error bars.

(c) The same analysis for a bigger concentration of random pinned regions (up to 10%) does not show significant changes of the above values of the exponents. This indicates that very likely when $p \neq 0$ and less of the percolation threshold there is a single universality class different from the $p=0$ case. We did not pursue our study up to the percolation threshold of the pinned regions since new phenomena, such as large scale lake formations, occur and hence all relevant scales would be altered. This was beyond the objective of the present work. Nor did we fully analyze further values of p since we expect only *two* universality classes to be present (with or without disorder) in view of the large amount of computer time involved.

(d) We explicitly checked that random precipitations both constant in time and updated at each perturbation [step (3) above] do not significantly change the critical exponents in both the homogeneous and heterogeneous terrains considered here. This is consistent with rigorous results derived in Ref. [12] in a related context where random rainfall has been shown to be irrelevant.

In conclusion our results indicate that real rivers are not well described by the critical exponents of the homogeneous model, i.e., $p=0$. However, by introducing a (small) concentration of quenched nonerodible regions, a new universality class is found which agrees well with the observational data. These pinned regions surrogate the ubiquitous presence of geomorphological constraints in real landscapes and thus their important role is appealing.

Enlightening discussions with Jayanth Banavar, Marek Cieplak, Francesca Colaiori, and Alessandro Flammini are acknowledged.

[1] For an overview see, e.g., I. Rodriguez-Iturbe and A. Rinaldo, *Fractal River Basins: Chance and Self-Organization* (Cambridge University Press, Cambridge, England, 1997), and references therein.

[2] R. Leheny and S.R. Nagel, Phys. Rev. Lett. **71**, 1470 (1993); A. Giacometti, A. Maritan, and J. Banavar, *ibid.* **75**, 577 (1995); J.R. Banavar, F. Colaiori, A. Flammini, A. Giacometti, A. Maritan, and A. Rinaldo (unpublished).

[3] T. Sun, P. Meakin, and T. Jössang, Phys. Rev. E **49**, 4865 (1994) and references therein.

[4] I. Rodriguez-Iturbe, R.L. Bras, E. Ijjasz-Vasquez, and D.G. Tarboton, Water Resour. Res. **28**, 988 (1992); D.G. Tarboton, R.L. Bras, and I. Rodriguez-Iturbe, *ibid.* **24**, 1317 (1988); **26**, 2243 (1990); P. La Barbera and R. Rosso, *ibid.* **25**, 735 (1989); D.R. Montgomery and W.E. Dietrich, Nature (London) **336**, 232 (1988); Science **255**, 826 (1992); D.G. Tarboton, R.L. Bras, and I. Rodriguez-Iturbe, Water Resour. Res. **25**, 2037 (1989).

[5] P. Bak, C. Tang, and K. Wiesenfeld, Phys. Rev. Lett. **59**, 381 (1987); Phys. Rev. A **38**, 364 (1988).

[6] A. Rinaldo, I. Rodriguez-Iturbe, R. Rigon, E. Ijjasz-Vasquez, and R.L. Bras, Phys. Rev. Lett. **70**, 822 (1993).

[7] M.A. Summerfield, *Global Geomorphology* (Longman, Singapore, 1991); W.E. Dietrich, C. Wilson, D.R. Montgomery, S.L. Reneau, and R. Bauer, J. Geol. **20**, 675 (1992).

[8] A. Maritan, A. Rinaldo, R. Rigon, A. Giacometti, and I. Rodriguez-Iturbe, Phys. Rev. E **53**, 1510 (1996).

[9] We have verified that, for samples of linear size $L=64$, averages up to 30 initial configurations do not change the final estimates, within the error bars.

[10] A value of $H=1/2$ would correspond to a network constructed according to a simple random-walk rule often referred to as the Scheidegger model in this context [see, e.g., H. Takayasu, Phys. Rev. Lett. **63**, 2563 (1989)].

[11] Self-similar river networks ($H=1$, $d_l>1$), have been discussed in M. Cieplak, A. Giacometti, A. Maritan, A. Rinaldo, I. Rodriguez-Iturbe, and J.R. Banavar (unpublished). It is noteworthy that any fractal network with stream-length exponent d_l yields identical exponents of a self-affine network with H if $d_l=2/(1+H)$ and vice versa.

[12] A. Maritan, F. Colaiori, A. Flammini, M. Cieplak, and J.R. Banavar, Science **272**, 984 (1996).